Remarks on Mr. Winthrop's paper on the duplication of the Cube in part 1ſt of this volume. By GEORGE BARON, *late Maſter of the Mathematical Academy at South Shields, in the County of Durham, in England.*

MR. Winthrop has attempted the duplication of the Cube, in a paper inſerted at page 25, part I. vol. 2, of the Memoirs of the American Academy of Arts and Sciences. This paper is entitled " Geometrical methods of finding any required ſeries of mean proportionals between given extremes." It conſiſts of four problems. In problem 1. is ſhewn the well known method of finding one mean proportional between two given extremes. Problem 2. is " To find two mean proportionals between two given extremes." On the truth of this problem the two remaining problems, together with Mr. Winthrop's duplication of the Cube, entirely depend. I ſhall therefore here, firſt deſcribe Mr. Winthrop's method of finding two mean proportionals between two given extremes: Secondly, ſhew that the demonſtration which he has attempted to give of that method is not true. And thirdly, I ſhall demonſtrate that that method is univerſally falſe. And hence it will follow, that Mr. Winthrop's duplication of the Cube is univerſally falſe.

Firſt then, I am to deſcribe Mr. Winthrop's method of finding two mean proportionals between two given extremes. The ſubſtance of it is as follows.

Let

Plate 1.

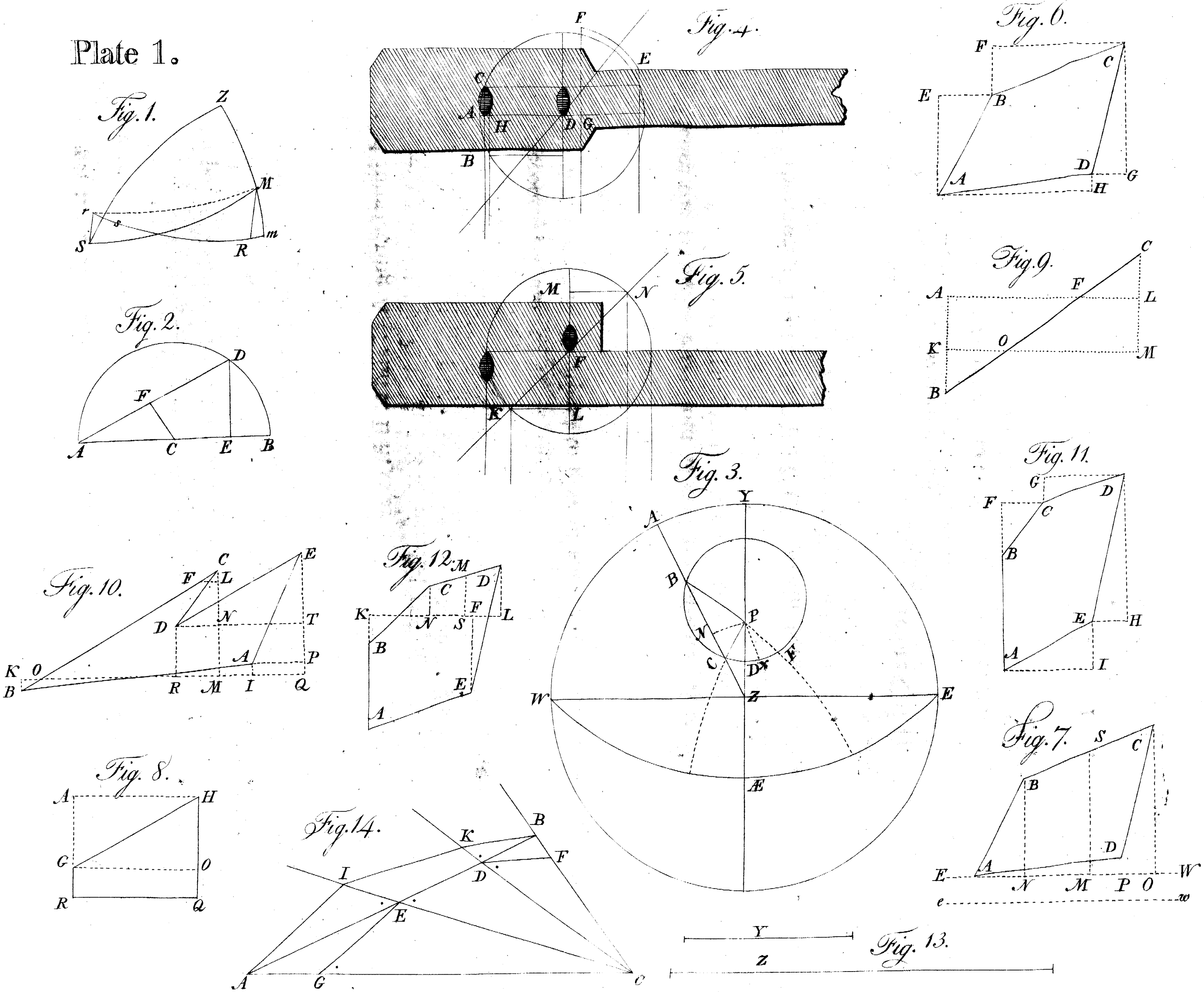
Fig. 1.
Z
M
r
s
S
R
m
Fig. 2.
D
F
A
C
E
B
Fig. 4.
F
E
C
A
H
D
G
B
Fig. 5.
M
N
F
K
L
Fig. 6.
F
E
B
C
A
D
H
G
Fig. 9.
C
A
F
L
K
O
M
B
Fig. 10.
C
F
L
E
D
N
T
K
O
A
P
B
R
M
I
Q
Fig. 12.
M
C
D
K
F
L
N
S
B
E
A
Fig. 3.
Y
A
B
P
N
C
D
F
W
Z
E
Æ
Fig. 11.
G
F
D
C
B
E
H
A
I
Fig. 7.
S
C
B
D
E
A
W
N
M
P
O
e
w
Fig. 8.
A
H
G
O
R
Q
Fig. 14.
B
K
F
I
D
E
A
G
c
Y
Z
Fig. 13.

Let it be required to find two mean proportionals between any two given extremes Y and Z. (Plate 1, fig. 13th.)

Take any ſtraight line A C=Z, (Plate 1, fig. 14) and from the point C draw C I making any angle A C I, leſs than one third of two right angles. Make the angles I C K and K C B each equal A C I, and make C B=Y. Join A B interſecting I C and K C in E and D. From E and D draw E G and D F, making the angles A E G and B D F each equal A C I, and interſecting A C and C B in G and F. Through A and B draw A I parallel to G E, and B K parallel D F, meeting C E and C D produced in I and K. Then are C K and C I the mean proportionals required.

Secondly, I am to ſhew that the demonſtration, which Mr. Winthrop has attempted to give of the above method, is not true. The ſubſtance of it is as follows.

The angle B D C=D E C+E C D; but B D F=E C D. Hence the triangles F D C and D E C are ſimilar. Alſo the angle A E C=E D C+D C E, but A E G=D C E: hence the triangles G E C, E D C and D F C are all ſimilar to each other. And therefore C F : C D : C E : C G. Again becauſe A I is parallel to G E, and K B parallel to D F, join I K, and I K is parallel to E D. Conſequently the triangles A C I, I C K and K C B are all ſimilar one to another; and therefore C B (Y) : C K : C I : C A (Z) as was required.

But although A I is parallel to G E, and K B is parallel to D F, it does not follow that I K is parallel to E D; and therefore Mr. Winthrop's demonſtration is not true.

I am thirdly to demonſtrate that Mr. Winthrop's method of performing this problem is univerſally falſe.

Becauſe Mr. Winthrop has not proved that I K is parallel to E D, it does not follow that I K is not really parallel to E D. Imagine therefore that I K is parallel to E D. Then will the triangles A I E, K B D, G E C, E D C, D F C, A I C, I K C and K B C be all ſimilar to one another. Therefore I E : K D :: C E : C D; and alſo I E : K D :: A I : B D. But A I is to B D in the complicated ratio of A I to K B and of K B to B D, and A I : K B :: A C : K C :: $\overline{CE}^2 : \overline{CD}^2$; alſo K B : B D :: C E : C D. Therefore A I : B D :: $\overline{CE}^3 : \overline{CD}^3$. But it has juſt been proved that A I : B D :: I E : K D :: C E : C D; and hence $\overline{CE}^3 : \overline{CD}^3$ :: C E : C D, which is abſurd. I K cannot therefore be parallel to E D; and conſequently this method of performing the problem is univerſally falſe. And as Mr. Winthrop's mode of duplicating the Cube depends upon the truth of this problem, it is alſo univerſally falſe. Q. E. D.

The ſame may be demonſtrated by a variety of other methods. For inſtance by the addition of ſines, tangents, &c. or by the logarithmic ſpiral.

I muſt at the ſame time acknowledge that Mr. Winthrop's miſtake on this ſubject was ſo natural, that I at firſt followed him in it. A ſecond inſpection convinced me of my miſtake.

Hallowell, in the District of Maine, June 2, 1798.

N. B. Soon after the paper, to which Mr. Baron's obſervations relate, was communicated to the Academy, it was examined by ſeveral of the members, ſkilled in mathematical ſcience, who were of opinion, that the author had not demonſtrated his theorem. But at the particular requeſt of the author the committee for publication conſented to its inſertion in the Memoirs.